

Distribution Costs and International Business Cycles

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Abstract

Backus, Kehoe, and Kydland (International Real Business Cycles, JPE, 100(4), 1992) documented several discrepancies between the observed post-war business cycles of developed countries and the predictions of a two-country, complete-market model. The main discrepancy dubbed as the quantity anomaly, that cross-country consumption correlations are higher than that of output in the model as opposed to data, has remained a central puzzle in international economics. In order to resolve this puzzle mainly two strategies: restrictions on asset trade, and introducing non-traded goods in the model, have been employed by researchers. While these extensions have been successful in closing the gap to some extent, the ordering of correlations has stayed unchanged: consumption correlations still exceed that of output. This paper attempts to resolve the quantity puzzle by introducing non-traded distribution costs in the retailing of traded goods. In a simple two-good extension of the Backus, Kehoe, and Kydland model, it is shown that without distributions costs the cross-country consumption and output correlations are 0.55 and 0.30, respectively, whereas with distribution costs consumption correlation reduces to 0.09, output correlation to 0.23. Incorporating distribution costs, in addition, improves the models performance in matching the volatility of real exchange rates and the correlation of net exports with output. These improvements are achieved without sacrificing the model's performance in any other dimension.

JEL classification codes: F32, F34, F41

Key words: open-economy business cycles; quantity anomaly; distribution costs; cross-country correlations.

1 Introduction

That cross-country consumption correlations substantially exceed output correlations in theoretical models, at complete variance with what is observed in the data for developed countries, has puzzled researchers in international economics for past two decades. The puzzle first encountered and dubbed as “quantity anomaly” by Backus, Kehoe, and Kydland (1992) emerged from a model world economy comprising of two countries, a single consumption good, and complete financial markets. In order to resolve this puzzle, subsequently, researchers have employed various strategies including restricting asset trades and introducing non-traded goods.¹ While these modifications have been successful in closing the gap to some extent, the *ordering* of the correlations has stayed unchanged: consumption correlations still exceed that of output. Table 1 summarizes the relative success of some of the past studies in in this respect.

This paper shows that extending the model of Backus et al. (1992) to include non-traded goods and distribution services resolves the quantity anomaly by accomplishing the right ordering of cross-country quantity correlations. Distribution services are understood as the services required to make a unit of traded goods *consumable* and entails transportation, wholesaling, and retailing services, which are all produced in the non-traded sector of the economy.

The reason that the quantity anomaly arises in the model of Backus et al. (1992) is easy to understand. First, market completeness allows agents across countries to pool their consumption risk. Second, an efficient allocation of resources commands that labor and investment be relatively higher in the country with the higher productivity. As a result, while consumptions comove across countries, outputs move in the opposite directions.

A departure from the complete markets assumption then seems a natural first step to take. Baxter and Crucini (1995), however, show that asset market structure matters only when the shocks are near unit root. In particular, the model’s predictions do not

¹Incorporating multiple sectors into the model is another strategy: see Ambler et al. (2002), Kouparitsas (1998), and Huang and Liu (2003). For models that incorporate multiple countries, see Yakhin (2004) and Burstein and Tesar (2005).

improve significantly under trend stationary shocks when asset trade is restricted to a sole riskless bond. Intuitively, with trend-stationary shocks the permanent component of income changes are relatively small. As a result, the insurance role of asset markets is relatively less important than its role in facilitating the efficient allocation of resources across countries. The existence of a riskless bond still allows agents to time their production and consumption decisions efficiently. Under unit-root shocks on the other hand, all income changes are permanent and the insurance role of asset market dominates. Complete markets perfectly pool permanent income changes across countries and thus obtain a near-perfect consumption correlation. In a bond economy, the country with a positive shock increases its consumption relatively more than its output, while the other country raises its production and lowers its consumption. As a result cross-country output and consumption correlations switch signs relative to complete markets. In a similar vein, Kollman (1996) studies a bond economy with shocks that are more persistent (autocorrelation: 0.95) than those studied by Backus et al. (1992) (autocorrelation: 0.91) and obtains a significantly lower cross-country consumption correlation relative to other studies. Kehoe and Perri (2002) endogenize market incompleteness by introducing ‘enforcement constraints’. They also assume highly persistent shocks as in Kollman (1996) and rule out cross-country spillovers. This improves the ordering of quantity correlations quite substantially. Finally, Heathcote and Perri (2002) completely eliminates asset trade. Yet their model’s consumption correlation far exceeds the output correlation.

Stockman and Tesar (1995) attack the puzzle by undertaking a different line of research: instead of relaxing the assumption of complete markets, they disaggregate production and consumption into traded and non-traded sectors. To understand how this approach helps, consider a positive shock to the non-traded goods sector in one of the countries. Insofar as the increased production of non-traded goods must be entirely consumed within the country, the consumption of non-traded goods increases and, by the tradable-nontradable consumption complementarity, it calls for a rise in the consumption of traded goods. As a result, both output and consumption in one country rise without a rise in the consumption of the

Table 1: Cross-country correlation of consumption and output in previous and present research

	$\rho(c, c^*)$	$\rho(y, y^*)$
Backus et. al. (1992)	0.88	-0.18
Baxter and Crucini (1995)	0.92 (−0.28)	0.06 (0.54)
Kehoe and Perri (2002)	0.29	0.25
Kollman (1996)	0.38	0.10
Heathcote and Perri (2002)	0.85	0.24
Stockman and Tesar (1995)	0.68	0.63
Present work	0.15	0.26

Notes: figures in the table are the best predictions of the corresponding models. For Baxter and Crucini (1995), the number without (within) brackets corresponds to a bond economy under shocks that are trend-stationary (unit-root without cross-country spillovers).

other. Thus, introducing non-traded goods significantly helps in aligning consumption and output comovement. However, the model solely driven by technology shocks obtains near-unity cross-country correlation of the traded consumption. Authors then incorporate taste shocks in the model by allowing relative preference for traded consumption to differ across countries and across time stochastically. This lowers consumption correlations for obvious reasons, but, lacking a sound economic rationale, taste shocks are hard to comprehend.

The success of the past studies in closing the gap between cross-country quantity correlations (denoted by ρ) is summarized by Table 1. It bears emphasis that in all the models except the bond economy with unit-root shocks, $\frac{\rho(c, c^*)}{\rho(y, y^*)} > 1$, i.e., the cross-country correlation of consumption exceeds that of output.

The main thesis of this paper is that when the standard two-country model with complete financial markets is extended to include distribution services, the model produces the correct ordering of the cross-country correlations of consumption and output. Moreover, the results are obtained with standard trend-stationary shocks. To underscore the role of distribution costs, an effort has been made to depart as little as possible from the canonical complete-markets model used by Backus et al. (1992). Including distribution services in an otherwise standard two-country model requires, however, adding a non-traded sector to the economy as in Stockman and Tesar (1995). But unlike Stockman and Tesar (1995) who further disaggregate traded goods into importables and exportables, the model presented

here confines itself to a single traded good.² Finally, once again to emphasize the role of distribution services, most of the functional forms and model parameters from Backus et al. (1992) and Stockman and Tesar (1995) are utilized while calibrating the model in section 3.

In recent years distribution costs have drawn much interest from researchers in explaining several international relative price puzzles. In a two-country framework Burstein, Neves, and Rebelo (2003) explain the ‘excess’ real exchange rate appreciations during exchange-rate-based stabilizations plans by introducing distribution costs. They also conjecture that distribution costs may potentially explain the quantity puzzle. In their paper, “distribution margin” is defined as the difference between retail and producer price as a percentage of retail price. They find a distribution margin of 43.4% for traded goods consumption in the US. For other G-7 countries it ranges from 35% in France to 50% in Japan. Following Burstein et al. (2003) and other studies, it is assumed that distribution services expend a fixed amount of non-traded goods per unit of traded good retailed to consumers.³

In a standard two-good model traded output and traded consumption, by definition, are identical goods. With distribution costs, traded output and consumption are two distinct entities as each unit of final traded consumption good incorporates a non-traded component in addition to a unit of traded good. Thus, effectively, the model with distribution costs can be viewed as a model without distribution costs but with a modified utility function that has a substantially stronger complementarity between traded and non-traded goods.⁴ This alternative interpretation helps in explaining the model’s improved performance in the following dimensions.

First, consider first a shock to the non-traded sector in the home country. As already shown by Stockman and Tesar (1995), introducing non-traded goods restores the connection between output and consumption movements. With distribution costs, a stronger

²Nevertheless, the shock process estimated by Stockman and Tesar (1995) fits the two-country, two-sector model presented in the next section, and is therefore utilized for simulating the model.

³See also Corsetti and Dedola (2005) examine pass-through to import and consumer prices in a New-Keynesian model of international price discrimination.

⁴In the model, the increased complementarity depends on the distribution parameter – the amount of non-traded output required per unit of traded output in retailing a unit of traded consumption.

implied complementarity between the consumption of the two goods calls for a relatively higher demand for traded goods. Thus, insofar as non-traded shocks in one country break the consumption commovements, with distribution costs the impact is stronger. With sufficiently large shocks to the non-traded productivity in home, while foreign may still produce more of traded goods to supply increased demand at home, the consumption of both goods relatively increase more in the home country. Thus, while output comovement is largely unaffected, consumptions move in opposite directions. This, in addition, improves the model's performance in reproducing countercyclical trade balance. Now consider a shock to the traded sector at home. In a standard two-good model the windfall in the traded sector is shared across countries, which requires that non-traded production also increase in both. With distribution costs, however, the increase in demand for non-traded goods is higher. Insofar as cross-sectoral productivities are more correlated than cross-country traded productivities, the consumption benefits of the increased traded productivity may largely remain localized. Again, the performance of the model in reproducing consumption-output comovement and countercyclical trade balance improves.

A standard two-good model produces a volatility of real exchange rate which is much lower when compared to the data.⁵ The model with distribution costs performs better in this dimension as well.⁶ The intuition behind this is the following. Consider a positive shock to the traded-sector in any one country. A standard two good-model, due to the usual complementarity of consumption, will call for an increase in the relative price of non-traded goods. With distribution costs the increase will be larger due to a substantially stronger implied complementarity between the two goods. Similarly, a decline in the relative price of non-traded goods under positive shocks to non-traded productivity will be steeper under distribution costs. As price index directly depends on the relative price of non-traded goods, the real exchange rate (the ratio of price levels) is more volatile.

The remainder of the paper is organized as follows. The model is presented in the

⁵See Obstfeld and Rogoff (2000) who describe it as one of “the six major puzzles in international economics”.

⁶Craighead (2004) finds that real exchange rate is more volatile when real rigidities - distribution services along with imperfect mobility in labor markets - are incorporated into the model.

Section 2, while its functional specification, steady state computation, and calibration is performed in Section 3. Section 4 presents numerical results, while in Section 4 the model's impulse responses are discussed. Section 5 offers some concluding remarks.

2 The model

The model below extends the two-country complete-market model of Backus et al. (1992) to include non-traded goods.⁷ It is assumed that delivering traded goods to consumers requires distribution services, which utilize non-traded goods as its sole input. In particular, retailing a unit of traded goods to consumer requires ψ units of non-traded goods to be spent in distribution services. The cost of distributing traded goods introduces a wedge between the producer and the consumer price of traded goods: while the law of one price across countries holds at the producer level, the non-traded component involved in the distribution prevents the law to hold at the consumer level.

The world economy consists of two symmetric countries in the tradition of Lucas' (1982), countries that are indexed by 1 and 2. The symmetry across countries allows to economize on notation by describing only the problem faced by country 1; the problem of country 2 is identical. Each country is populated by an infinitely large number of identical households. The representative household derives utility from leisure, ℓ_t , and consumption of a composite good, c_t , consisting of traded and non-traded goods, respectively c_t^T and c_t^N :

$$c_t = \hat{c}(c_t^T, c_t^N)$$

where $\hat{c}(c_t^T, c_t^N)$ is a linear homogeneous aggregator function, increasing and concave in both sectorial consumptions. The household is endowed with a unit of time that is allocated between leisure, ℓ_t , and the production of traded and non-traded goods, respectively n_t^T

⁷Unlike Stockman and Tesar (1995) who disaggregate traded goods further into exportables and importables, here the choice of a single homogenous traded good helps in identifying the role of distribution costs in resolving the quantity puzzle more sharply.

and n_t^N :

$$\ell_t \leq 1 - n_t^T - n_t^N \quad (1)$$

The representative household maximizes the following utility function:

$$W = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \alpha(L) \ell_t), \quad (2)$$

where $\alpha(L)$ is the lag operator $\alpha(L) = \sum_{m=0}^{\infty} \alpha_m L^m$ used by Kydland and Prescott (1982) to generate intertemporal substitution of leisure. Further, $\sum_{m=0}^{\infty} \alpha_m = 1$, and $\alpha_m = (1 - \eta)^{m-1} \alpha_1$ for $m \geq 1$, and where $\eta \in (0, 1]$.

Each country has access to two sectoral technologies, one for producing traded and one for producing non-traded goods. Traded goods are allocated between consumption, investment, and net exports, denoted respectively as c_t^T , x_t^T , and nx_t . By exploiting the equilibrium equivalence between aggregate per-capita variables and variables of the atomistic household, the following represents the aggregate constraint on the use of traded goods:

$$f^T(\lambda_t^T, k_t^T, n_t^T, z_t) = c_t^T + x_t^T + nx_t, \quad (3)$$

where f^T is a CES function that produces traded goods by the use of capital, labor, and inventories z_t , and which is perturbed by the productivity shock λ_t^T . Physical capital used in the production of traded goods is installed by investment projects initiated in the traded sector. As in Stockman and Tesar (1995), it is assumed that capital is sector specific, i.e., capital in the traded goods sector cannot be moved to produce output in the non-traded goods sector and vice versa.

Non-traded goods are produced by using capital, labor, and land. Land is assumed to be in fixed in the aggregate.⁸ The aggregate constraint on the use of non-traded goods is as follows:

$$f^N(\lambda_t^N, k_t^N, n_t^N, \bar{h}) = c_t^N + \psi c_t^T + x_t^N \quad (4)$$

⁸The inclusion of land in the production function of non-traded goods follows Burstein et al. (2003).

where f^N represents a CES production function of non-traded goods whose inputs are capital k_t^N , labor n_t^N , and land \bar{h} ; λ_t^N is a sector-specific productivity shock. Non-traded goods are used for direct consumption as well as distributing traded goods to consumers: a unit of output of traded goods becomes consumable after spending ψ units of non-traded goods. Additionally, output of non-traded goods is used for investment in physical capital for the non-traded sector.

Capital takes *time to build*. In particular, it takes J^T (J^N) periods for an investment project in traded (non-traded) sector to mature. Thus

$$x_t^T = \sum_{j=1}^{J^T} \phi_j^T s_{jt}^T + z_{t+1} - z_t, \quad (5a)$$

$$x_t^N = \sum_{j=1}^{J^N} \phi_j^N s_{jt}^N \quad (5b)$$

where ϕ_j^i denotes the fraction of investment expenditure allocated to raise the stock of capital j -periods into the future, and s_j^i denotes the total outlay on a capital project in sector i , j periods before the project's maturity. In other words, $\phi_j^i s_{jt}^i$ is the date t expenditure on a capital project in sector i that matures into a capital stock of s_j^i on date $t+j$. Equation (5a) specifies that investment in the traded goods sector comprises additions to the stock of capital and changes in inventories, while (5b) states that investment in the non-traded sector is solely used for accumulating physical capital. The laws of motion of the sectoral capital stocks depend on sector-specific depreciation rates, δ^T and δ^N , and the number of sector-specific projects started $J^T - 1$ and $J^N - 1$ periods ago:

$$k_{t+1}^T = (1 - \delta^T) k_t^T + s_{1t}^T, \quad (5c)$$

$$k_{t+1}^N = (1 - \delta^N) k_t^N + s_{1t}^N, \quad (5d)$$

Technology shocks follow a VAR(1) process:

$$\lambda_{t+1} = \mathbf{\Lambda} \lambda_t + \varepsilon_t$$

where $\lambda_t \equiv [\lambda_{1t}^T, \lambda_{1t}^N, \lambda_{2t}^T, \lambda_{2t}^N]$ is a vector encompassing the productivities of the two industries in the two countries; $\mathbf{\Lambda}$ is a 4×4 -matrix of autorregressive coefficients; and $\varepsilon_t \equiv [\varepsilon_{1t}^T, \varepsilon_{1t}^N, \varepsilon_{2t}^T, \varepsilon_{2t}^N]$ is a vector of i.i.d. innovations to the process characterized by its variance-covariance matrix Σ .

An equivalent social planner's problem: Under complete markets, the allocations in a decentralized equilibrium are identical to that obtained by a social planner who maximizes the combined world welfare. The planner is free to move traded goods across countries but she has to observe the country-specific constraints on the use of non-traded goods.⁹ Formally, the planners' constraint in the traded goods sector is given by

$$\sum_{i=1,2} f^T(\lambda_{it}^T, k_{it}^T, n_{it}^T, z_{it}) = \sum_{i=1,2} (c_{it}^T + x_{it}^T) \quad (6)$$

where subscript i denotes variables related to country i . Given the lifetime expected utility W defined in (2), the problem of a planner who cares equally about both countries consist in maximizing $W_1 + W_2$ subject to constraints (1), and (4) to (6). Note that (6) combined with (3) implies that

$$nx_1 = -nx_2, \quad (7)$$

i.e., in equilibrium, inflows of traded goods in one country equal the outflows in the other.

$$x_t^T = \sum_{j=1}^{J^T} \phi_j^T s_{jt}^T + z_{t+1} - z_t, \quad (8a)$$

$$x_t^N = \sum_{j=1}^{J^N} \phi_j^N s_{jt}^N \quad (8b)$$

where ϕ_j^i denotes the fraction of investment expenditure allocated to raise the stock of capital j -periods into the future, and s_j^i denotes the total outlay on a capital project

⁹The ability of the planner to move goods across countries is equivalent, in a decentralized equilibrium, to the existent of state-contingent commitments between agents that will entail a flow of traded output across countries.

in sector i , j periods before the project's maturity. In other words, $\phi_j^i s_{jt}^i$ is the date t expenditure on a capital project in sector i that matures into a capital stock of s_j^i on date $t+j$. Equation (5b) specifies that investment in the traded goods sector comprises additions to the stock of capital and changes in inventories, while (5b) states that investment in the non-traded sector is solely used for accumulating physical capital. The laws of motion of the sectoral capital stocks depend on sector-specific depreciation rates, δ^T and δ^N , and the number of sector-specific projects started $J^T - 1$ and $J^N - 1$ periods ago:

$$k_{t+1}^T = (1 - \delta^T) k_t^T + s_{1t}^T, \quad (8c)$$

$$k_{t+1}^N = (1 - \delta^N) k_t^N + s_{1t}^N, \quad (8d)$$

Technology shocks follow a VAR(1) process:

$$\lambda_{t+1} = \mathbf{\Lambda} \lambda_t + \varepsilon_t$$

where $\lambda_t \equiv [\lambda_{1t}^T, \lambda_{1t}^N, \lambda_{2t}^T, \lambda_{2t}^N]$ is a vector encompassing the productivities of the two industries in the two countries; $\mathbf{\Lambda}$ is a 4×4 -matrix of autorregressive coefficients; and $\varepsilon_t \equiv [\varepsilon_{1t}^T, \varepsilon_{1t}^N, \varepsilon_{2t}^T, \varepsilon_{2t}^N]$ is a i.i. vector of innovations to the process characterized by its variance-covariance matrix $\mathbf{\Sigma}$.

3 Functional specification, steady state, and model calibration

3.1 Functional forms

The aggregator function $\hat{c}(c_t^T, c_t^N)$ is a CES function similar to the one adopted by Stockman and Tesar (1995):

$$\hat{c}(c_t^T, c_t^N) = \left[\omega (c_t^T)^{-\rho} + (1 - \omega) (c_t^N)^{-\rho} \right]^{-\frac{1}{\rho}} \quad (9a)$$

where ω and $1 - \omega$ are the weights in agents' preferences of traded and non-traded goods and $(1 + \rho)^{-1}$ is the elasticity of substitution between traded and non-traded goods. The specification of the utility index and the production function of traded goods follow the specification of the utility index and production function of the aggregate output in Backus et al. (1992):

$$U(c_t^T, c_t^N, \ell_t) = \gamma^{-1} \left\{ [\hat{c}(c_t^T, c_t^N)]^\mu [\alpha(L) \ell_t]^{1-\mu} \right\}^\gamma \quad (9b)$$

$$f^T(\lambda_t^T, k_t^T, n_t^T, z_t) = \left[\left(\lambda_t^T (k_t^T)^{\theta^T} (n_t^T)^{1-\theta^T} \right)^{-\nu^T} + \sigma z_t^{-\nu^T} \right]^{-\frac{1}{\nu^T}} \quad (9c)$$

In (9b), μ is a consumption-leisure share parameter and $(1 - \gamma)^{-1}$ is the intertemporal elasticity of substitution; in (9c) $(1 + \nu^T)^{-1}$ is the elasticity of substitution between inventories and the capital-labor aggregate.

The specification of the production function in the non-traded sector is borrowed from Burstein et al. (2003):

$$f^N(\lambda_t^N, k_t^N, n_t^N, \bar{h}) = \left[\pi \left(\lambda_t^N (k_t^N)^{\theta^N} (n_t^N)^{1-\theta^N} \right)^{-\nu^N} + (1 - \pi) \bar{h}^{-\nu^N} \right]^{-\frac{1}{\nu^N}}, \quad (9d)$$

where $(1 + \nu^N)^{-1}$ denotes the elasticity of substitution between the sector specific capital-labor aggregate and land.

3.2 Steady State

Given the preferences and technology specified above, at the steady, the optimality conditions involving capital, inventories, and land are:¹⁰

$$q^T(r + \delta^T) = \theta^T \left(\frac{y^T}{k^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right], \quad (10a)$$

$$r = \sigma \left(\frac{y^T}{z} \right)^{1+\nu^T}, \quad (10b)$$

$$q^N(r + \delta^N) = \theta^N \left(\frac{y^N}{k^N} \right) \left[1 - (1 - \pi) \left(\frac{y^N}{\bar{h}} \right)^\xi \right], \quad (10c)$$

¹⁰A technical appendix that derives first-order conditions and steady state equations is available from the authors upon request.

where $r = \beta^{-1} - 1$ equals the steady-state real interest rate and q^T and q^N , are the shadow prices of capital in the traded and non-traded sector, which are defined as:

$$\begin{aligned} q^T &= \frac{1}{J^T} \sum_{j=1}^{J^T} (1+r)^{j-1}, \\ q^N &= \frac{1}{J^N} \sum_{j=1}^{J^N} (1+r)^{j-1}, \end{aligned}$$

Eq. (10a) indicates that the gross rate of return on capital, $q^T (r + \delta^T)$, must equal the marginal product of capital. Similarly, (10b) states that the opportunity cost of inventories, r , must equal its marginal product. Equation (10c) is similar to (10a) when adapted to the technology in the non-traded sector.

The optimality conditions for the supply of labor and its utilization in traded and non-traded sectors imply:

$$\frac{1 - \theta^N}{\theta^N} q^N (r + \delta^N) \frac{k^N}{n^N} = \frac{1 - \mu}{\mu} \left[\frac{\alpha r + \eta}{r + \eta} \right] \frac{\left[1 + \frac{\omega}{1-\omega} \left(\frac{c^N}{c^T} \right)^\rho \right] c^N}{1 - n^T - n^N}, \quad (11)$$

$$\frac{\omega}{1 - \omega} \left(\frac{c^N}{c^T} \right)^{1+\rho} = \frac{q^N (r + \delta^N)}{q^T (r + \delta^T)} \frac{k^N}{n^N} \frac{n^T}{k^T} \frac{1 - \theta^N}{\theta^N} \frac{\theta^T}{1 - \theta^T} + \psi, \quad (12)$$

Eq. (11) characterizes the standard labor-leisure choice and equates the marginal product of labor in the non-traded sector with marginal rate of substitution of leisure for consumption of non-traded goods. The choice of labor between the traded and the non-traded sector yields equation (12) which equates the marginal rate of substitution between traded and non-traded goods to the marginal rate of transformation between the two goods. Notice that the first term on the RHS denotes the ratio of the marginal products of labor in the two sectors. As wages across sectors are equal due to perfect labor mobility, this ratio (implicitly) equals the relative price of traded goods in terms of non-traded goods at the *producer* level. But, before a unit of traded goods is *retailed* to the consumer, additional ψ units of non-traded goods are used by the distribution services. Hence, the RHS of (12) represents the *consumer price* of traded goods in terms of non-traded goods.

At the steady state, investment equals depreciation in each of the two sectors and inventory investment is equal to zero. Further, in a symmetric stationary equilibrium the net flow of goods across countries equals to zero and the market for non-traded goods clear:

$$\begin{aligned} c^T &= y^T - \delta^T k^T \\ c^N &= y^N - \psi c^T - \delta^N k^N \end{aligned}$$

3.3 Model calibration

Distribution costs The data obtained from the National Accounts of OECD Countries indicates that the consumption of traded goods is approximately equal to the consumption of non-traded goods. On the production side, traded output constitutes only one third of non-traded output. This signifies that the retailing of traded goods incorporate non-traded components. First normalize the relative price $p^N \equiv P^N/\bar{P}^T = 1$. To calibrate the distribution cost parameter ψ , note that in the model the quantity of traded consumption $c^T = y^T - \delta^T k^T$. In the data, the measured traded consumption expenditure is $\tilde{c} \equiv c^T + \psi c^T$, where ψc^T reflects the distribution expenditure as per the specification of the model (since $p^N = 1$). Using the measured ratio of traded consumption expenditure to traded output and the model's traded sector resource constraint, one obtains the following expression:

$$\frac{1}{1 + \psi} \frac{\tilde{c}^T}{y^T} + \delta \frac{k^T}{y^T} = 1$$

which solved for ψ gives $\psi = 1.023$. This implies a distribution margin equal to 51% which is close to 44% as reported by by Burstein et al. (2003) for the US economy.

Preferences Following Stockman and Tesar (1995), the elasticity of substitution between traded and non-traded goods in consumption is set equal to $(1 + \rho)^{-1} = 0.44$ (i.e. $\rho = 1.27$). It follows from (9b) that the elasticity of substitution between consumption and leisure is unity, which is consistent with the empirical observation that despite a substantial growth in real wages the share of time allocated to work has remained constant over time. Finally,

as in Backus et al. (1992) and Stockman and Tesar (1995), $\gamma = -1$, which implies an intertemporal elasticity of substitution $(1 - \gamma)^{-1} = 0.5$, which is well within the range of values used in the business-cycle literature. The benchmark exercise sets $\alpha = 1$ to eliminate distributed lags on leisure. For the remaining exercises when $\alpha \neq 1$, $\eta = 0.1$ is set following Backus et al. (1992). The share parameter $\mu = 0.35$ is obtained from the marginal condition between non-traded consumption and leisure, in equation (11). The value of the relative weight parameter on the traded consumption, $\omega = 0.27$, arises from condition (12).

Technology Following Backus et al. (1992) (respectively Burstein et al. (2003)), the elasticity parameter ν^T (ν^N) in the traded (nontraded) goods production function is set equal to 3 (2). For calibrating the other parameters in the traded-sector technology, following Backus et al. (1992), the volume of inventories is assumed to equal output. Then, (10b) implies $\sigma = 0.01$. In the data, the share of labor in the traded sector is approximately 0.55, which, given the share of inventories equal to 1%, obtains a value of 0.44 for θ^T . In the data, the share of labor in the non-traded sector is 0.58. Following Burstein et al. (2003) the share of land in non-traded output is taken as 5%. This gets a value for θ^N approximately equal to 0.39

The marginal conditions in the traded and non-traded sectors, equations (10a) and (10b), imply a capital/output ratios of approximately 10 in both sectors, which are consistent with the data and past studies. Following Backus et al. (1992), it is assumed that the average time allocated to market activities is 30%. Accordingly $n \equiv n^T + n^N = 0.3$. Then, given the labor shares of output in the two sectors, and given the output ratios, the values of n^T and n^N arise in a straightforward manner; $n^T = 0.072$, and $n^N = 0.228$.

Equating the ratio of marginal products of land and labor to the ratio of rental rate on land and wages in the decentralized equilibrium yields

$$\frac{1 - \theta^N}{1 - \pi} \left[(y^N)^{-\nu^N} - (1 - \pi) \bar{h}^{-\nu^N} \right] = \frac{wn^N}{\varrho \bar{h}} \quad (13)$$

In the expression above, ϱ denotes the rental on land and w the wage rate. After normalizing

the quantity of land to unity, i.e., $\bar{h} = 1$, one obtains $\pi = 0.99$.

Finally, the productivity parameters λ^T and λ^N , are chosen so that the ratio of traded to non traded output is consistent with the data, which is approximately equal to 1/3. The model parameter values are summarized in Table 2.

The shock process The shock process is borrowed from Stockman and Tesar (1995), which is reproduced below:

$$\mathbf{\Lambda} = \begin{bmatrix} 0.154 & 0.040 & -0.199 & 0.262 \\ -0.150 & 0.632 & -0.110 & 0.125 \\ -0.199 & 0.262 & 0.154 & 0.040 \\ -0.110 & 0.125 & -0.015 & 0.632 \end{bmatrix}$$

As noted by Stockman and Tesar (1995), the autorregression matrix shows that the shocks in non-traded sector are more persistent than in the traded sector. Notice that the autoregression matrix is symmetric across countries. The variance-covariance matrix estimated by Stockman and Tesar (1995) is reproduced below

$$\mathbf{\Sigma} = \begin{bmatrix} 3.62 & 1.23 & 1.21 & 0.51 \\ 1.23 & 1.99 & 0.51 & 0.27 \\ 1.21 & 0.51 & 3.62 & 1.23 \\ 0.51 & 0.27 & 1.23 & 1.99 \end{bmatrix}$$

Some features of the variance-covariance matrix are noteworthy. First, the variance of shocks is relatively larger in the traded than in the non-traded sector (compare Σ_{11} with Σ_{22}). Second, the cross-sectoral within-country correlations are larger than the cross-country correlations in the traded sector (compare $\Sigma_{12}/\sqrt{\Sigma_{11}\Sigma_{22}}$ with Σ_{13}/Σ_{11}). The non-traded sector across countries show a very low correlation, while cross-country cross-sectoral correlations are slightly larger.

3.4 The consumer price index and the real exchange rate

The discussion in the next section will refer to the consumption-based price index and the real exchange rate of the economies in the model, so it is useful to derive them here.

Let \bar{P}^T and P^N denote the price of traded and non-traded goods at the producer level in a common unit of account. Further, denote the consumer price of traded goods as $P^T \equiv \bar{P}^T + \psi P^N$, where the identity follows from the assumption of perfect competition in the distribution sector. Then the consumer price index (CPI) can be derived as

$$P = \left[\omega^{\frac{1}{1+\rho}} (P^T)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} (P^N)^{\frac{\rho}{1+\rho}} \right]^{\frac{1+\rho}{\rho}}$$

Then, using the definition of P^T and P^N , the CPI, in terms of traded goods at the producer level, can be derived easily as

$$\frac{P}{\bar{P}^T} = \left[\omega^{\frac{1}{1+\rho}} (1 + \psi p^N)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} (p^N)^{\frac{\rho}{1+\rho}} \right]^{\frac{1+\rho}{\rho}}, \quad (14)$$

where $p^N = \frac{P^N}{\bar{P}^T}$ is the relative price of non-traded goods relative to the producer price of traded goods.¹¹ By definition, the law of one price holds for traded goods at the producer level, which is normalized to one. Assuming that the nominal exchange rate is equal one, the real exchange rate of economy 1 is equal to the ratio of the price indexes:

$$\text{RER} = \left[\frac{\omega^{\frac{1}{1+\rho}} (1 + \psi p_2^N)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} (p_2^N)^{\frac{\rho}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} (1 + \psi p_1^N)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} (p_1^N)^{\frac{\rho}{1+\rho}}} \right]^{\frac{1+\rho}{\rho}}$$

where the subscripts 1 and 2 denote the relative prices in country 1 and 2, respectively.

4 Quantitative results

This section presents the quantitative implications of extending the model in Backus et al. (1992) to include nontradable goods and distribution services. The discussion starts study-

¹¹In other words, p^N is the relative price of non-traded goods in terms of traded goods at the factory.

ing the within-country business cycle implications and then shifts focus to the cross-country correlations. All reported model statistics are obtained by simulating the economy 300 times using 300 periods in each simulation and disregarding the dynamics of the first 200 periods. Impulse response functions are obtained from once-and-for-all shocks to the productivity of the domestic industries equal to the square root of the elements Σ_{11} and Σ_{22} of the covariance matrix of the shocks. Impulse-response functions, therefore, incorporate the technology spillovers implicit in the VAR process shown in Section 3, but do not take into consideration the covariance of the innovations.

4.1 Distribution costs and domestic business cycles

Table 3 compares the standard deviations and correlations with output predicted by a model that has only one good with the predictions of a model in which there are traded and non-traded goods. The table also shows the effects of adding distribution services to the consumption of tradable goods in the two-good economy. And to provide a benchmark for the comparisons, Table 3 also reproduces the US business cycle statistics reported by Backus et al. (1992) in their Table 1.

Columns 1 to 3 of Table 3 contains the US estimates. Columns 4 to 6 correspond to the one-good economy, and are taken from Table 4 of Backus et al. (1992). The next three columns correspond to the economy with two goods in which the law of one price holds for tradable goods, i.e $\psi = 0$; and the last three columns show business cycle statistics of the economy in which that law does not hold due to the nontradable distribution services that are required to convert tradable goods into consumable tradable goods.

A known result in the literature is that the one-good model with complete markets exaggerates the volatility of investment: while the empirical relative volatility of investment in column 2 is equal to 3.15, the relative volatility of investment predicted by the one-good model is equal to 10.94 (col. 5). As can be seen in the table (see col. 8), adding non-traded goods to the two-country model reduces the volatility of investment which becomes closer to its empirical counterpart. However, the introduction of non-traded goods reduces by

half the volatility of consumption. When the law of one price does not hold for traded goods, the two-good model with distribution services improves the theoretical prediction of the volatility of consumption and preserves the data-like volatility of investment. What the two-good model, either with or without distribution services, fails to replicate is the correlation between the trade balance and output since it implies the trade balance is procyclical.¹²

To understand the role of distribution services on domestic business cycles, consider first a productivity shock to the production of non-traded goods in one of the countries and, for a moment, assume away the distribution services. The effect of the shock is depicted by dashed impulse-response functions in Figure 1. Sectorial output and consumption, shown in panels a), b), d) and e) of the table, are measured in physical units while their corresponding aggregates in panels c) and f) are measured in units of traded goods using the relative price of non-traded consumption goods shown in panel k). The shock raises the supply of non-traded goods which in turn lowers the relative price of these goods and appreciates the real exchange rate. As domestic goods become cheaper, domestic agents revise the composition of their optimal consumption basket: whereas their consumption of non-traded goods increases by more than 30% on impact, the consumption of traded goods only raises by less than 15%. On the production side of the economy, labor is reallocated towards the production of non-traded goods, the sector in which labor is marginally more productive; likewise, most investment projects seek to increase the stock of capital in non-traded industries rather than in industries producing traded goods. On impact, the fall in the production of traded goods accompanied by a higher domestic consumption of these goods creates a gap on the domestic availability of tradable goods that is closed by a fall in the number of investment projects started in the traded industry. Because of the time-to-build technology, the fall in the number of projects can be seen through the decline in

¹²The reason why the two-good model of this paper fails to match the countercyclical trade balance observed in practice, whereas the model in Stockman and Tesar (1995) is successful in this margin is due to the way traded goods are modeled in each case. Whereas there is only one traded good in the model of this paper, there are two in their model. With two traded goods, the raise in domestic consumption of traded goods will raise the demand for both importable and exportable goods, turning the trade balance countercyclical.

output of traded goods only after 4 periods have elapsed. Thus, net exports rise before they become negative.

The impulse-response functions that follow the same shock to the production of non-traded output in the model with distribution services are represented by solid lines in Figure 1. Panels a) and b) reveal that adding distribution services to the model does not change much the response of the supply side of the economy. On the demand side, however, the relative abundance of non-traded goods created by the productivity shock also reduces the cost of consuming traded goods. Consumption of traded and non-traded goods, measured in physical units, rise now by roughly the same proportion in both sectors. Interestingly, aggregate consumption, measured in units of traded goods at retail level, falls. This is because, in the case of non-traded goods, the negative price negative dominates the positive quantity effect. As explained in the next subsection, this response of aggregate consumption lowers the cross-country correlation of consumption. As panel k) shows, the relative price of non-traded goods falls more in the economy with distribution services making that the price effect, instead of the quantity effect, becomes dominant to make aggregate consumption fall instead of rise.

Consider now a shock to the productivity in the sector that produces traded goods; the effect of the shock is shown in Figure 2. Shocks to the productivity of the traded goods sector are less persistent than shocks affecting the productivity of the sector that produces non-traded goods and this explains why macroeconomic aggregates are close to the stationary value around period 10. The shock raises the production of tradable goods, labor is reallocated toward the production of traded output and fewer investment projects are started to raise the capital stock in the industry producing non-traded output. The relative price of non-traded goods increases more when the consumption of traded goods requires to use distribution services because the higher consumption of traded goods raises the demand for non-traded output indirectly through a higher demand for these services. This explains why the real exchange rate depreciates more on impact in the economy with distribution services and why aggregate consumption increases more in this economy than

in the economy where the law of one price holds for traded goods.

The fact that productivity shocks to the traded-good sector are larger than the shocks to the non-traded-good sector and the fact that aggregate consumption raises more when the law of one price does not hold for traded goods explains why consumption is more volatile when distribution services are added to the model. As can be seen in Figures 1 and 2, the introduction of these services has an almost negligible effect on the volatility of output, something consistent with the results shown in Table 3.

4.2 Distribution costs and international correlations

Focus changes here to the effects of introducing non-traded goods and distribution costs on the cross country correlations of output and consumption. The main finding of this paper is to show that when distribution services are necessary to convert traded goods into consumable traded goods, a two-country, two-good model with complete markets solves the quantity anomaly documented by Backus et al. (1992) and succeeds at aligning theory and data regarding the relative value of the aforementioned cross country correlations. Table 4 summarizes the finding. There is shown that whereas the cross-country correlation of consumption exceeds the cross-country correlation of output in the standard two-country, one good model as well as in the two-good model without distribution services, the opposite happens in the economy with distribution services. In particular, while in the data the ratio of the cross-country correlation of consumption to the cross-country correlation of output is equal to 0.34, this ratio has the opposite sign and an absolute value higher than one (4.89) in the one-good model of Backus et al. (1992). The two-good model without distribution costs generates a ratio with the correct sign although still higher than one (it is equal to 1.83). With distribution services, the two-good model produces a ratio equal to 0.52, which is close to its empirical counterpart.

Figure 3 shows the relationship between the value of the distribution cost parameter ψ and the ratio of cross country correlations discussed above. Results in the figure are

obtained re-calibrating the model for each value of ψ .¹³ The values of ψ considered in the graph are between 0 and the value used in the calibration of the precedent section equal to 1.0623. When distribution services are ignored, i.e. $\psi = 0$, the ratio of cross country correlations is positive; only when the distribution-cost parameter exceeds 0.63 the cross-country correlation of aggregate consumption falls below the cross-country correlation of aggregate output.

To explain why distribution services succeed at solving the quantity anomaly, Figures 4 and 5 display impulse responses of output and consumption for the two countries in the model. As before, solid lines are used to identify the outcome of the model with distribution services and dashed lines are used to identify results of the model that excludes these services; additionally, to distinguish between the home and foreign country, starred lines are used for the latter. In Figure 4, one of the countries, let say the home country, receives a positive productivity shock in its traded-good industry. Complete financial markets make efficient to have the home country enlarging the production of tradable goods while reducing the production of non-traded output. A similar pattern of production is observed abroad; at the same time, consumption increases in both countries. Overall, there is nothing in Figure 4 that permits to understand why the cross-country correlation of consumption is lower than the cross-country correlation of output when distribution costs are added to the model and the economy is hit by shocks to the production of traded goods. Attention should then focus to shocks in the sector producing non-traded goods.

Figure 5 depicts impulse response functions for the two countries when a positive shock hits the domestic industry of non-traded goods. As explained before, in period 1, when spillover effects do not come along yet, the domestic production of traded goods falls and the domestic production of non-traded goods rises. Abroad, output in both industries, measured in physical units, is almost invariant in period 1. Despite this, there is a hike in consumption in the foreign country regardless the value of the distribution cost parameter

¹³The re-calibration exercise is done as follows. As in the data, it is assumed that the consumption expenditure in the two sectors is almost equal. Then, a value of ψ is assumed. This essentially affects the output ratios in the two sectors. The rest of the calibration exercise is done as before.

that is due to the existence of complete markets. More akin to the main finding of this paper are the results of panels c) and f) of the figure. Panel c) shows that aggregate output in the two countries are *negatively* correlated so the positive cross country correlation of output in Table 3 must be explained by the shocks to traded-good industries discussed above. As shown in Table 3 and this Figure 5, distribution services do not change much the correlation of output across countries; as panel c) of Figure 4 makes clear, distribution services explain why consumptions in the two countries do not co-move much. When the law of one price holds for traded goods, aggregate consumption move in the same direction in both countries. Once distribution services are added to the model, aggregate consumption, measured in units of tradable output falls at home. This happens in spite of the units of each good consumed increase in both countries (see panels d) and e)) and permits to conclude that the negative correlation of consumption across countries following a shock to the non-traded industry is mostly explained by the behavior of the relative price of non-traded goods.

5 Conclusions

Equilibrium business cycle theory has been successful at accounting for the fluctuations and co-movement of the main macroeconomic aggregates of modern market economies. When the theory was extended to evaluate how global financial markets could alter the equilibrium decisions of optimizing agents and consequently the behavior of the macroeconomic aggregates, theorists found that business-cycle models endowed with complete financial markets have a hard time explaining the observed cross-country correlations of output and consumption. In particular, theory predicts that consumption should be more correlated than output because countries are able to insure away their aggregate macroeconomic risk. As the opposite happens in practice, Backus et al. (1992) dubbed this theory-data inconsistency as the quantity anomaly.

Several theories aimed at solving this anomaly have been proposed since then with partial success. Most successful attempts attacked the problem by relaxing the assumption of

complete financial markets and by including taste shocks into the models. The achievement at solving the anomaly has been, notwithstanding, partial: in all cases, consumption remains more correlated than output across countries. This paper proposes an alternative theory to explain the anomaly that does not rely on relaxing the complete market assumption. The proposed theory adds a non-traded goods sector to the economy and recognizes that traded goods become consumable only after they are combined with wholesale and retail distribution services. That distribution services are an important component of the value of traded goods is empirically well backed up (See Burstein et al. (2003)).

When a country is hit by a positive productivity shock in the production of non-traded goods, the relative price of non-traded goods falls in equilibrium so as to raise the domestic absorption of non-traded goods. The quantities consumed of both traded and non-traded goods rise because they are complement goods. When distribution services are assumed away, aggregate consumption in the two countries, measured in units of traded goods, co-move after the shock. Introducing distribution services reverts this result and consumption in the two countries become negatively correlated. This happens because the domestic relative price of non-traded goods falls more sharply now; consequently, the price effect dominates the quantity effect and aggregate consumption falls at home while it raises in the foreign country. Furthermore, a stronger complementarity of traded and non-traded goods along with the shock process that exhibits a stronger cross-sectoral correlation relative to the cross-country correlations is able to restore the connection between GDP and consumption movements relatively strongly than in the model without distributions shocks.

Finally, the model with distribution costs is also able to improve the contemporaneous correlation between trade balance and output. However, the trade balance is still strongly procyclical. A potential avenue to rectify this counterfactual feature would be to introduce exportables and importable into the model as in Stockman and Tesar (1995). This is left for further exploration.

6 Appendix: Solution to the planner's problem: steady states

Below, we solve the planner's problem in order to derive symmetric steady states. The planner's problem is to maximize $W_1 + W_2$, where W_1 and W_2 denote the present discounted value of the representative agents utility in country 1 and 2 respectively, where

$$W \equiv \sum_{t=0}^{\infty} \beta^t U(c_t^T, c_t^N, \alpha(L)(1 - n_t - n_t^N))$$

The set of constraints each country faces is

$$\begin{aligned} c_t^T + x_t^T + nx_t &= f^T(k_t^T, n_t^T, z_t^T), \\ \psi c_t + c_t^N + x_t^N &= f^N(k_t^N, n_t^N, \bar{h}), \\ x_t^T &= \sum_{j=1}^{J^T} \phi_j s_{jt}^T + z_{t+1} - z_t, \\ x_t^N &= \sum_{j=1}^{J^N} \phi_j^N s_{jt}^N, \\ k_{t+1}^T &= (1 - \delta) k_t^T + s_{1t}^T, \\ k_{t+1}^N &= (1 - \delta^N) k_t^N + s_{1t}^N, \end{aligned}$$

In addition, $nx_1 = -nx_2 = 0$ holds at a symmetric steady state; then it suffices to focus on the Lagrangian component of any one country. To do so, substitute the first four constraints into the utility and write the Lagrangian function as follows,

$$\begin{aligned} \mathcal{L}_t &= \sum_{m=0}^{\infty} \beta^m U \left(\begin{aligned} &f^T(k_{t+m}^T, n_{t+m}^T, z_{t+m}^T) - \sum_{j=1}^{J^T} \phi_j^T s_{jt+m}^T - z_{t+m+1} + z_{t+m} - nx_{t+m}, \\ &f^N(k_{t+m}^N, n_{t+m}^N, \bar{h}) - \psi \left(\begin{aligned} &f^T(k_{t+m}^T, n_{t+m}^T, z_{t+m}^T) \\ & - \sum_{j=1}^{J^T} \phi_j^T s_{jt+m}^T - z_{t+m+1} + z_{t+m} \end{aligned} \right) - \sum_{j=1}^{J^N} \phi_j^N s_{jt+m}^N, \\ &\alpha(L)(1 - n_{t+m}^T - n_{t+m}^N) \end{aligned} \right) \\ &+ \sum_{m=0}^{\infty} \beta^m \chi_{t+m}^T ((1 - \delta^T) k_{t+m}^T + s_{1t+m}^T - k_{t+m+1}^T) + \sum_{m=0}^{\infty} \beta^m \chi_{t+m}^N ((1 - \delta^N) k_{t+m}^N + s_{1t+m}^N - k_{t+m+1}^N) \end{aligned}$$

where χ_t^T and χ_t^N are the multipliers (shadow prices) on the sector-specific capital accumulation equations in traded and non-traded sectors, respectively. Note that $s_{j,t+1} = s_{j+1,t}$. Noting that after substituting $c^N = y^N - \psi c^T - x^N$, c^T occurs in both the first and the second argument of the utility function, define $U_{c^T} \equiv U_1 - \psi U_2$. Then, the first order conditions can be written as (for convenience we assume $\alpha(L) = 1$ for now; we will revisit lags at the time of deriving steady states)

$$\begin{aligned}
s_{Jt}^T &: \phi_J U_{c^T}(c_t^T, c_t^N, l_t) + \beta \phi_{J-1} U_{c^T}(c_{t+1}^T, c_{t+1}^N, l_{t+1}) + \dots + \beta^{J-1} \phi_1 U_{c^T}(c_{t+J-1}^T, c_{t+J-1}^N, l_{t+J-1}) - \beta^{J-1} \chi_{t+J}^T \\
s_{Kt}^N &: \phi_K U_2(c_t^T, c_t^N, l_t) + \beta \phi_{K-1} U_2(c_{t+1}^T, c_{t+1}^N, l_{t+1}) + \dots + \beta^{K-1} \phi_1 U_2(c_{t+K-1}^T, c_{t+K-1}^N, l_{t+K-1}) - \beta^{K-1} \chi_{t+K}^N \\
k_{t+1}^T &: \beta f_1^T(k_{t+1}^T, n_{t+1}^T, z_{t+1}) U_{c^T}(c_{t+1}^T, c_{t+1}^N, l_{t+1}) - \chi_t^T + \beta(1 - \delta^T) \chi_{t+1}^T = 0, \\
k_{t+1}^N &: \beta f_1^N(k_{t+1}^N, n_{t+1}^N, \bar{h}) U_2(c_{t+1}^T, c_{t+1}^N, l_{t+1}) - \chi_t^N + \beta(1 - \delta^N) \chi_{t+1}^N = 0, \\
z_{t+1} &: \beta f_3^T(k_{t+1}^T, n_{t+1}^T, z_{t+1}) U_{c^T}(c_{t+1}^T, c_{t+1}^N, l_{t+1}) - U_{c^T}(c_t^T, c_t^N, l_t) + \beta U_{c^T}(c_{t+1}^T, c_{t+1}^N, l_{t+1}) = 0, \\
n_t^T &: U_3(c_t^T, c_t^N, l_t) = f_2^T(k_t^T, n_t^T, z_t) U_{c^T}(c_t^T, c_t^N, l_t). \\
n_t^N &: U_3(c_t^T, c_t^N, l_t) = f_2^N(k_t^N, n_t^N, \bar{h}) U_2(c_t^T, c_t^N, l_t) = f_2^N(k_t^T, n_t^T, z_t) U_{c^T}(c_t^T, c_t^N, l_t). \text{ (from above)}
\end{aligned}$$

Steady state After dropping the arguments of the functions for notational convenience, the steady-state version of the first order conditions can be rewritten as:

$$\begin{aligned}
s_J^T &: \sum_{j=1}^{J^T} (1+r)^{j-1} \phi_j^T = \frac{\chi^T}{U_{c^T}} \equiv q^T \\
s_K^N &: \sum_{j=1}^{J^N} (1+r)^{j-1} \phi_k^T = \frac{\chi^N}{U_2} \equiv q^N \\
k^T &: f_1^T = \left(\frac{1}{\beta} - 1 + \delta^T \right) \frac{\chi^T}{U_{c^T}} = q^T (r + \delta^T) \\
k^N &: f_1^N = \left(\frac{1}{\beta} - 1 + \delta^N \right) \frac{\chi^N}{U_2} = q^N (r + \delta^N) \\
z &: f_3^T = \frac{1-\beta}{\beta} \equiv r \\
n^T &: f_2^T = \frac{U_3}{U_{c^T}} \\
n^N &: \frac{U_{c^T}}{U_2} = \frac{f_2^N}{f_2^T}, \text{ or } \frac{U_1}{U_2} = \frac{f_2^N}{f_2^T} + \psi
\end{aligned}$$

Additionally,

$$\begin{aligned}
c^T &= f^T(k^T, n^T, z) - \delta^T k^T \\
c^N &= f^N(k^N, n^N, \bar{h}) - \psi c^T - \delta^N k^N
\end{aligned}$$

must hold. This system of nine equations must be solved by the value of the following nine variables: $\bar{c}, \bar{c}^N, \bar{k}, \bar{k}^N, \bar{n}, \bar{n}^N$, and \bar{z} .

Functional forms and the steady state For future use note the following

$$\begin{aligned}
f_1^T &= \theta^T \left(\frac{y^T}{k^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right] \\
f_2^T &= (1 - \theta^T) \left(\frac{y^T}{n^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right]
\end{aligned}$$

and

$$\begin{aligned} f_1^N &= \theta^N \left(\frac{y^N}{k^N} \right) \left[1 - (1 - \pi) \left(\frac{y^N}{h} \right)^{\nu^N} \right] \\ f_2^N &= (1 - \theta^N) \left(\frac{y^N}{n^N} \right) \left[1 - (1 - \pi) \left(\frac{y^N}{h} \right)^{\nu^N} \right] \end{aligned}$$

The first-order conditions can be rewritten using the functional forms as

$$\begin{aligned} k^T &: f_1^T = \theta^T \left(\frac{y^T}{k^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right] = q^T (r + \delta^T) \\ z &: f_3^T = \frac{1 - \beta}{\beta}, \text{ or } r = \sigma \left(\frac{y^T}{z} \right)^{1 + \nu^T} \\ n^T &: \frac{U_3}{U_{c^T}} = \frac{U_3}{U_1 - \psi U_2} = f_2^T \implies \\ &= \frac{1 + \frac{1 - \omega}{\omega} \left(\frac{c^T}{c^N} \right)^\rho}{1 - \psi \left(\frac{1 - \omega}{\omega} \right) \left(\frac{c^T}{c^N} \right)^{1 + \rho}} \frac{1 - \mu}{\mu} \left[\frac{\alpha r + \eta}{r + \eta} \right] \frac{c^T}{1 - n^T - n^N} \\ &= f_2^T = (1 - \theta^T) \left(\frac{y^T}{n^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right] = q^T (r + \delta^T) \frac{1 - \theta^T}{\theta^T} \frac{k^T}{n^T} \\ k^N &: q^N (r + \delta^N) = \theta^N \left(\frac{y^N}{k^N} \right) \left[1 - (1 - \pi) \left(\frac{y^N}{h} \right)^{\nu^N} \right] \\ n^N &: \frac{U_{c^T}}{U_2} = \frac{f_2^N}{f_2^T}, \text{ or } \frac{U_1}{U_2} = \frac{f_2^N}{f_2^T} + \psi; \\ \text{or } \frac{\omega}{1 - \omega} \left(\frac{c^N}{c^T} \right)^{1 + \rho} &= \frac{(1 - \theta^N) \left(\frac{y^N}{n^N} \right) \left[1 - (1 - \pi) \left(\frac{y^N}{h} \right)^{\nu^N} \right]}{(1 - \theta^T) \left(\frac{y^T}{n^T} \right) \left[1 - \sigma \left(\frac{y^T}{z} \right)^{\nu^T} \right]} + \psi \\ &= \frac{q^N (r + \delta^N)}{q^T (r + \delta^T)} \frac{k^N}{n^N} \frac{n^T}{k^T} \frac{1 - \theta^N}{\theta^N} \frac{\theta^T}{1 - \theta^T} + \psi \\ &= \frac{\bar{P}^T}{\bar{P}^N} + \psi = p^{-1} + \psi \end{aligned}$$

where the last step in the first-order condition for n^N assumes that in a decentralized equilibrium the firm's optimization problem implies that the ratio of marginal products of labor in the two sector equal the ratio of producer prices given that the wage rate is equal in both sectors. The first order conditions with respect to k^T, z, n^T, k^N , and n^N are

compactly rewritten as (10a) - (12) in the main text.

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Table 2: Cross-country correlation of consumption and output in previous and the present research

	$\rho(c, c^*)$	$\rho(y, y^*)$
Backus et. al. (1992)	0.88	-0.18
Baxter and Crucini (1995)	0.92	0.06
	-0.28	0.54
Kehoe and Perri (2002)	0.29	0.25
Kollman (1996)	0.38	0.10
Heathcote and Perri (2002)	0.85	0.24
Stockman and Tesar (1995)	0.68	0.63
Present research	0.09	0.23

Notes: $\rho(c, c^*)$ and $\rho(y, y^*)$ denote, respectively, the cross-country correlations of consumption and output. Figures in the table are the best predictions of the corresponding models. Two set of results are reported for Baxter and Crucini's (1995) paper; although both correspond to the single-bond economy, the first set is for the trend-stationary shocks and the second set is for the unit-root shocks without cross-country spillovers.

Table 3: Benchmark parameter values; ADD EXPLANATION OF MEANING OF MODEL PARAMETERS

Preferences		
$\beta = 0.99$	$\omega = 0.27$	$\rho = 1.27$
$\mu = 0.35$	$\alpha = 1$	$\gamma = -1$
Technology (Traded)		
$\lambda^T = 1.42$	$\theta^T = 0.44$	$\nu^T = 3$
$\sigma = 0.01$	$\delta^T = 0.025$	$J^T = 4$
Technology (Non-traded)		
$\lambda^N = 1.97$	$\theta^N = 0.39$	$\nu^N = 2$
$\pi = 0.99$	$\delta^N = 0.025$	$J^N = 1$
	$\psi = 1.06$	

Table 4: Within-country business cycles statistics: volatilities and correlations with output

Variable	US Economy (Backus et.al. (1992))			One-Good Economy (Backus et.al. (1992)) Services			Two-Good Economy Without Distribution Services			Two-Good Economy With Distribution		
	Standard Deviation (1) %	(2) Relat.	Correlation w/output (3)	Standard Deviation (4) %	(6) Relat.	Correlation w/ Output (5)	Standard Deviation (7) %	(8) Relat.	Correlation w/ Output (9)	Standard Deviation (10) %	(11) Relat.	Correlation w/ Output (12)
Output												
Aggregate	1.71	1.00	1.00	1.55	1.00	1.00	2.03	1.00	1.00	1.95	1.00	1.00
Tradable	-	-	-	-	-	-	3.19	1.57	0.62	3.71	1.90	0.62
Nontradable	-	-	-	-	-	-	2.01	1.06	0.89	2.01	0.99	0.62
Consumption												
Aggregate	0.84	0.49	0.76	0.62	0.40	0.79	0.36	0.18	0.86	0.76	0.34	0.63
Tradable	-	-	-	-	-	-	0.33	0.16	0.92	0.57	0.29	0.69
Nontradable	-	-	-	-	-	-	0.42	0.21	0.75	0.77	0.39	0.58
Hours												
Aggregate	1.47	0.86	0.86	0.76	0.49	0.94	1.12	0.55	1.00	1.01	0.52	0.95
Tradable	-	-	-	-	-	-	2.42	1.19	0.80	3.16	1.62	0.50
Nontradable	-	-	-	-	-	-	1.46	0.72	0.22	1.19	0.61	0.64
Capital												
Aggregate	0.63	0.37	0.01	1.16	0.74	0.31	0.54	0.27	0.08	0.80	0.41	0.18
Tradable	-	-	-	-	-	-	0.89	0.44	0.11	1.75	0.90	0.13
Nontradable	-	-	-	-	-	-	0.42	0.21	-0.11	0.39	0.20	-0.18
Fixed Investment	5.38	3.15	0.90	16.91	10.94	0.27	7.01	3.45	0.44	8.18	4.20	0.37
Saving	-	-	-	-	-	-	6.37	3.14	1.00	6.16	3.16	0.97
Net Exports/Output	0.45	-	-0.28	2.90	-	-0.02	2.27	1.12	0.33	2.41	1.24	0.28
Real Exchange Rate	-	-	-	-	-	-	0.62	0.31	0.06	0.83	0.42	0.22

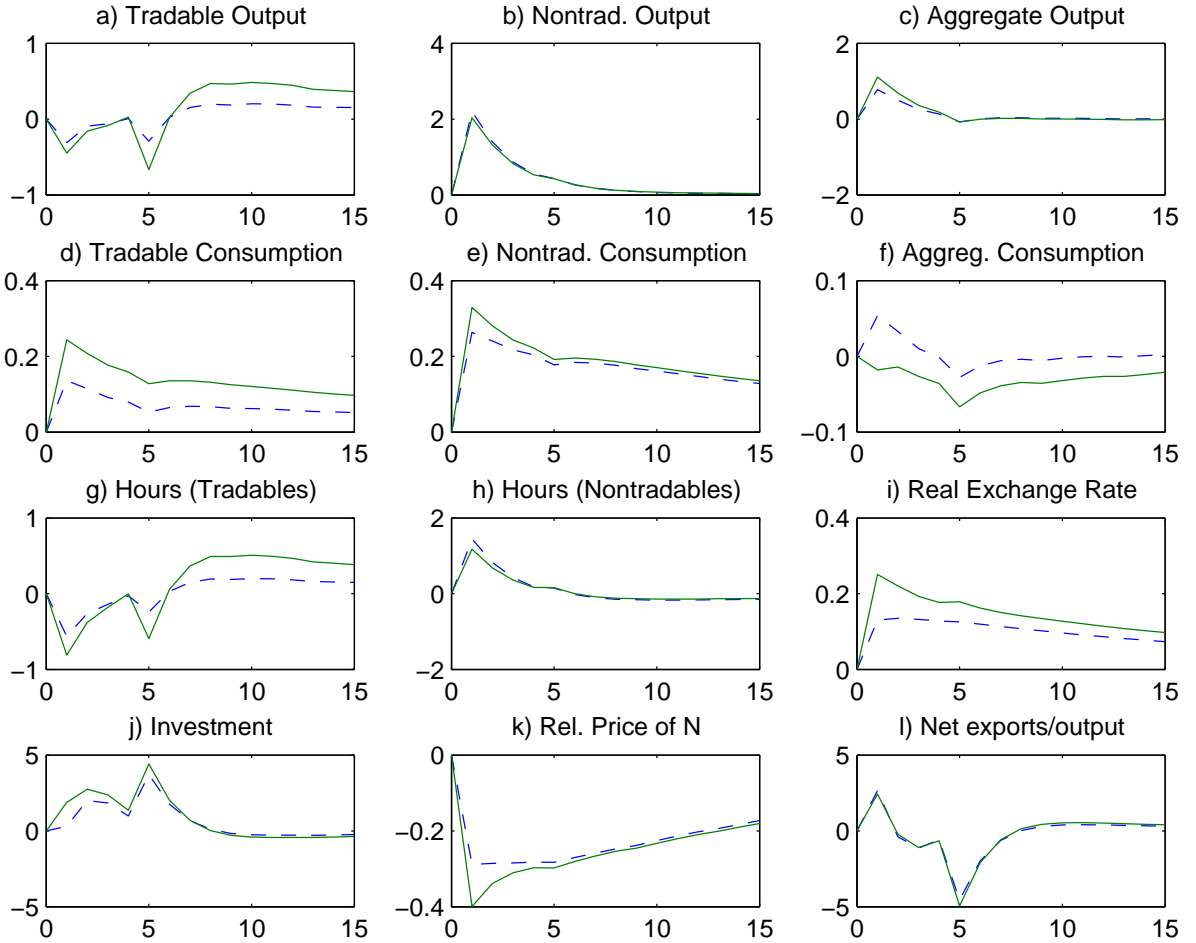
Notes: Columns (1) to (6) reproduce results in Tables 1 and 2 of Backus et al. (1992). Columns (7) to (9) correspond to the two-good economy without distribution services. Columns (10) to (12) correspond to the economy where distribution services are required to convert traded goods into consumable traded goods. Results in the last six columns were obtained simulating the corresponding models 300 times; each simulation was of length 300 but only the last 100 were considered to compute the mean of sample moments.

Table 5: Cross-country correlations

Variable	US Economy	One-Good Economy	Two-Good Economy	
			without distr. services	with distr. services
Foreign and Domestic Output				
Aggregate	0.56	-0.18	0.30	0.23
Tradable	-	-	0.20	0.02
Nontradable	-	-	0.13	0.11
Foreign and Domestic Consumption				
Aggregate	0.19	0.88	0.55	0.09
Tradable	-	-	0.54	0.12
Nontradable	-	-	0.51	0.07
Saving and Investment				
Real Exchange Rate	0.68	0.28	0.44	0.38
	-	-	0.25	0.21

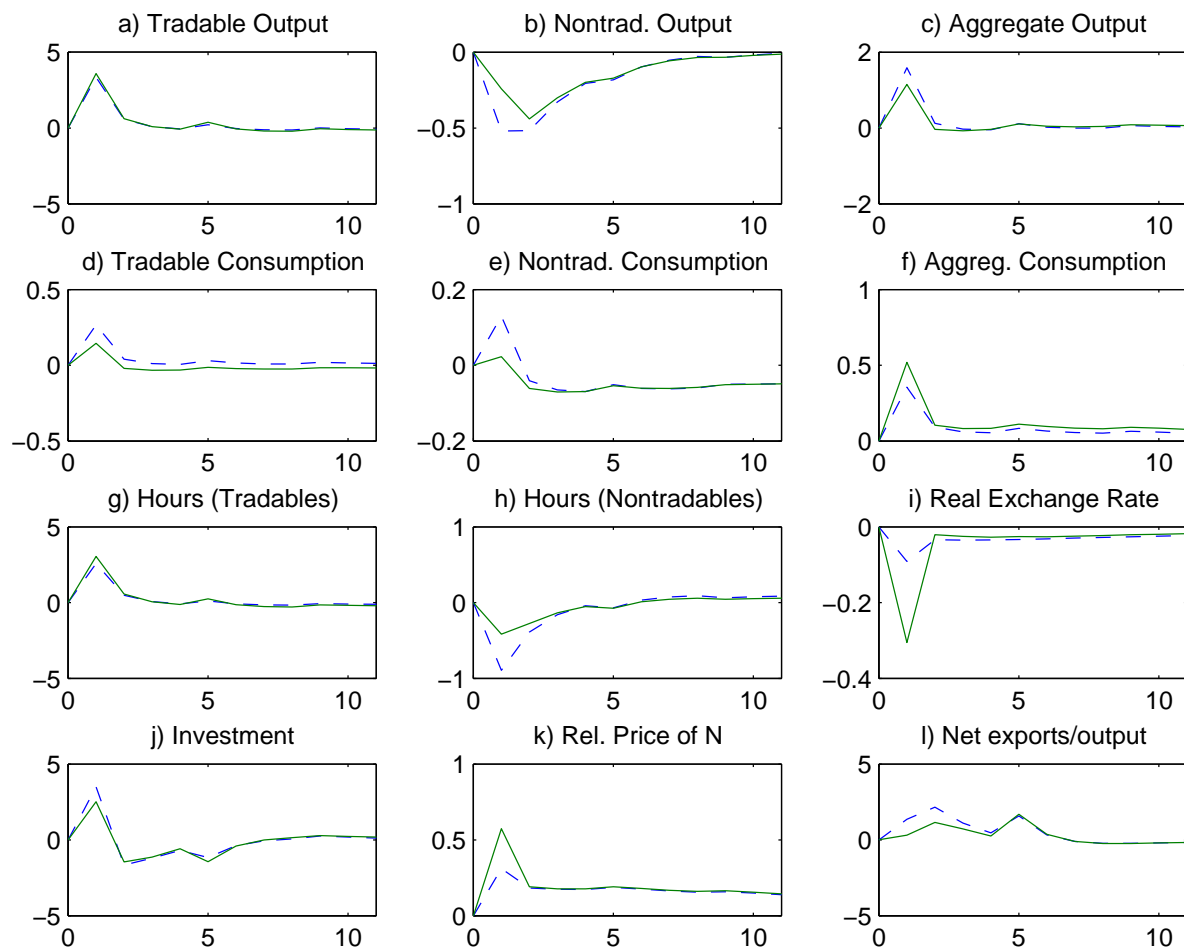
Notes: Data for the US Economy and the one-good model are taken from Backus et al. (1992); the empirical cross country correlations of output and consumption correspond to the average of the correlations between the US variable and the same variable in the group of 11 countries considered by these authors. Saving and Investment refer to within-country correlation of the domestic economy.

Figure 1: Effect of a productivity shock to the own production of non-traded goods with and without distribution services



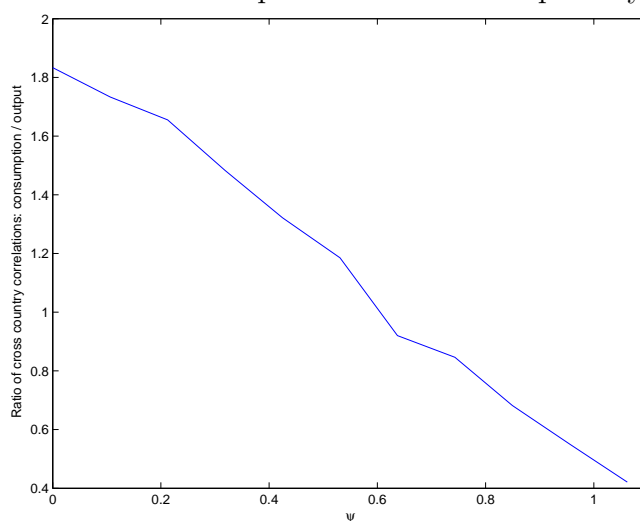
Note: Dashed lines are used for the economy without distribution services and solid lines are used for the economy with distribution services.

Figure 2: Effect of a productivity shock to the own production of traded goods with and without distribution services



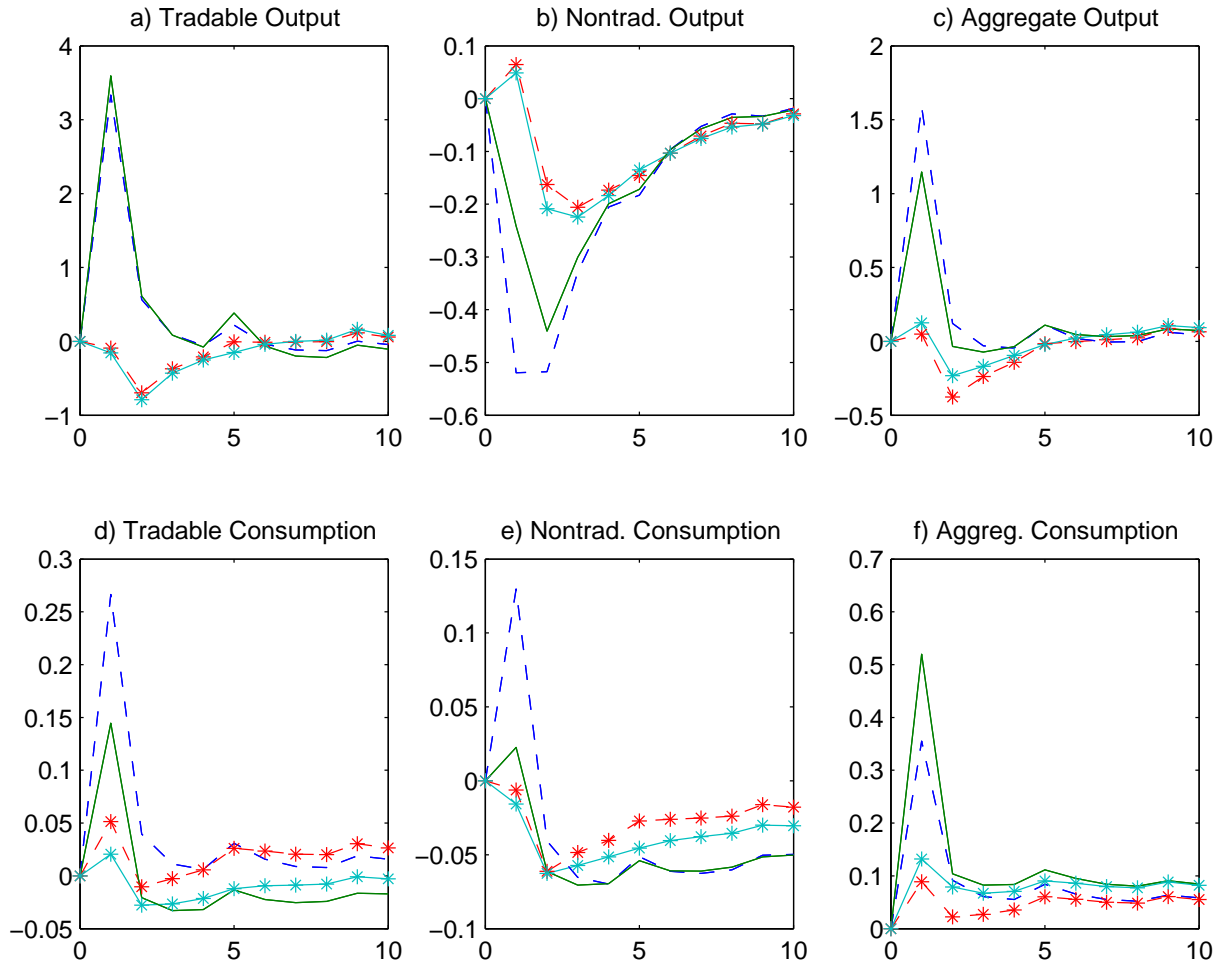
Note: Dashed lines are used for the economy without distribution services and solid lines are used for the economy with distribution services.

Figure 3: Distribution-cost parameter and the ‘quantity anomaly’



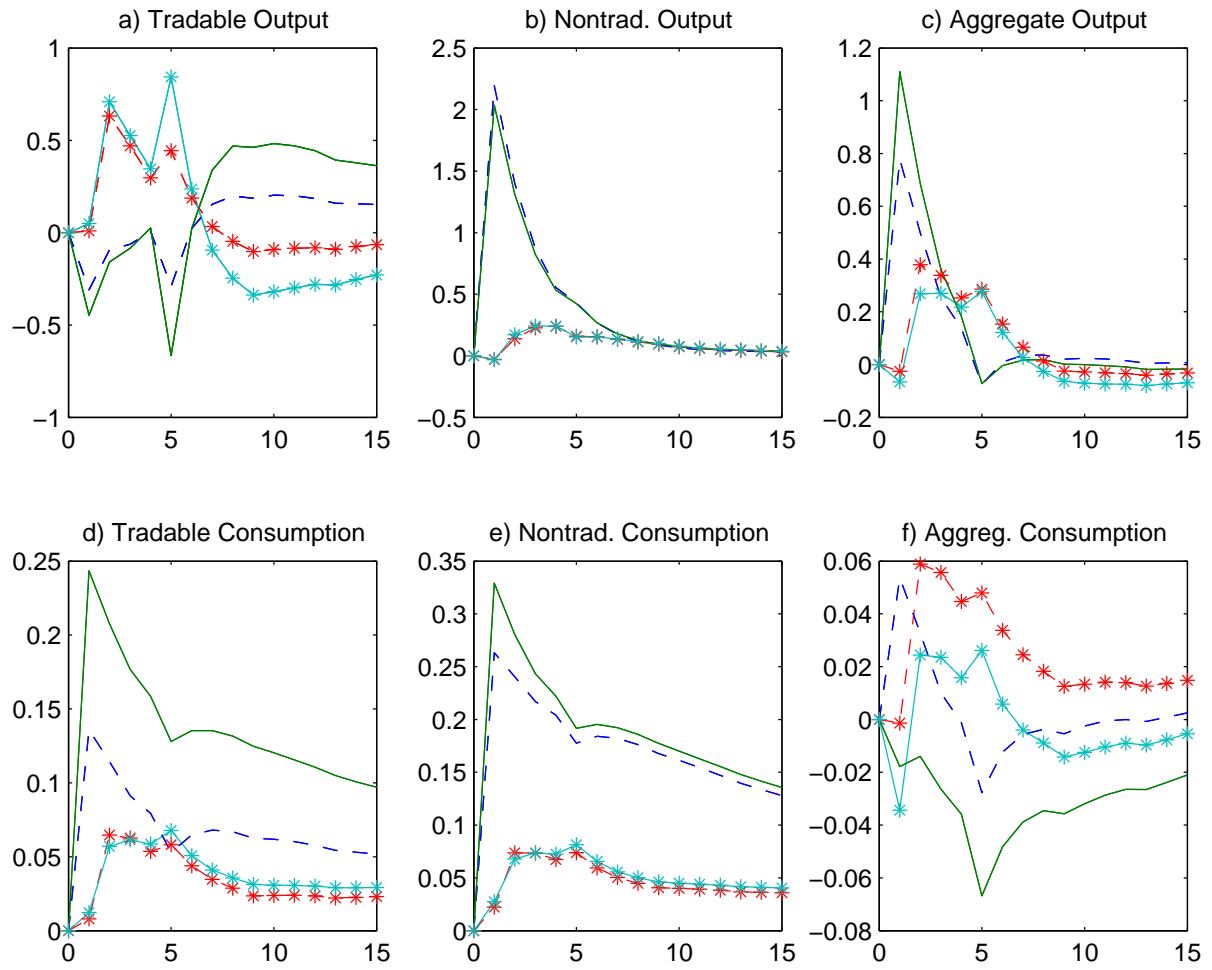
Note: Parameter ψ determines the size of the distribution margin; when $\psi = 0$ distribution services are not required to consume a unit of traded goods. $\psi = 1.06$ is the value used in the benchmark calibration of the model and indicates that the cost of a unit of consumable traded good is equal to

Figure 4: Cross-country effects of a shock to the production of traded goods at home



Note: Dashed lines are used for the economies without distribution services and solid lines are used for the economies with distribution services. Starred lines represent the variables of the foreign country.

Figure 5: Cross-country effects of a shock to the production of non-traded goods at home



Note: Dashed lines are used for the economies without distribution services and solid lines are used for the economies with distribution services. Starred lines represent the variables of the foreign country.